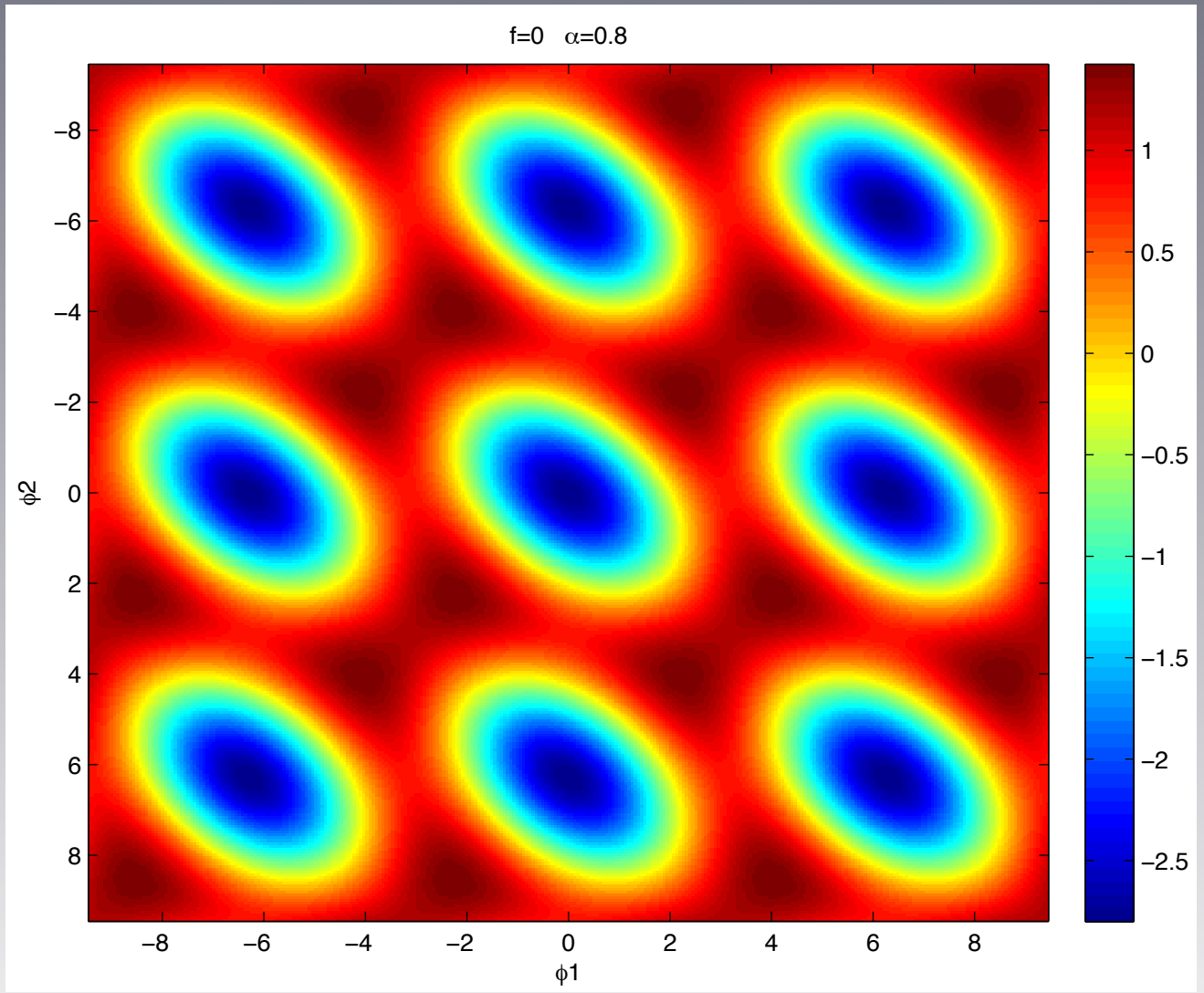
Quantum Information with Solid-State Devices

VO 141.246

Dr. Johannes Majer

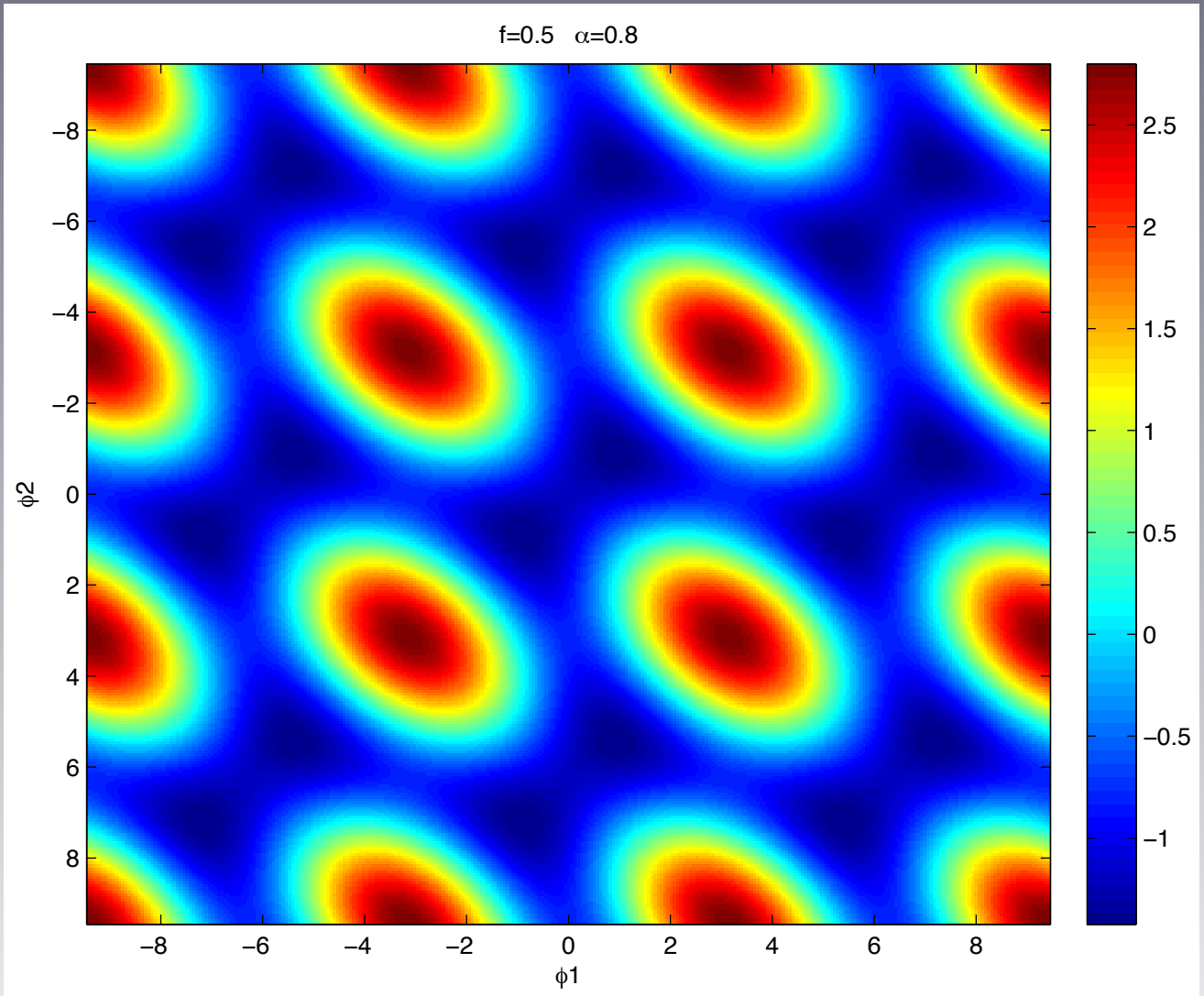
Lecture 10

persistent-current qubit



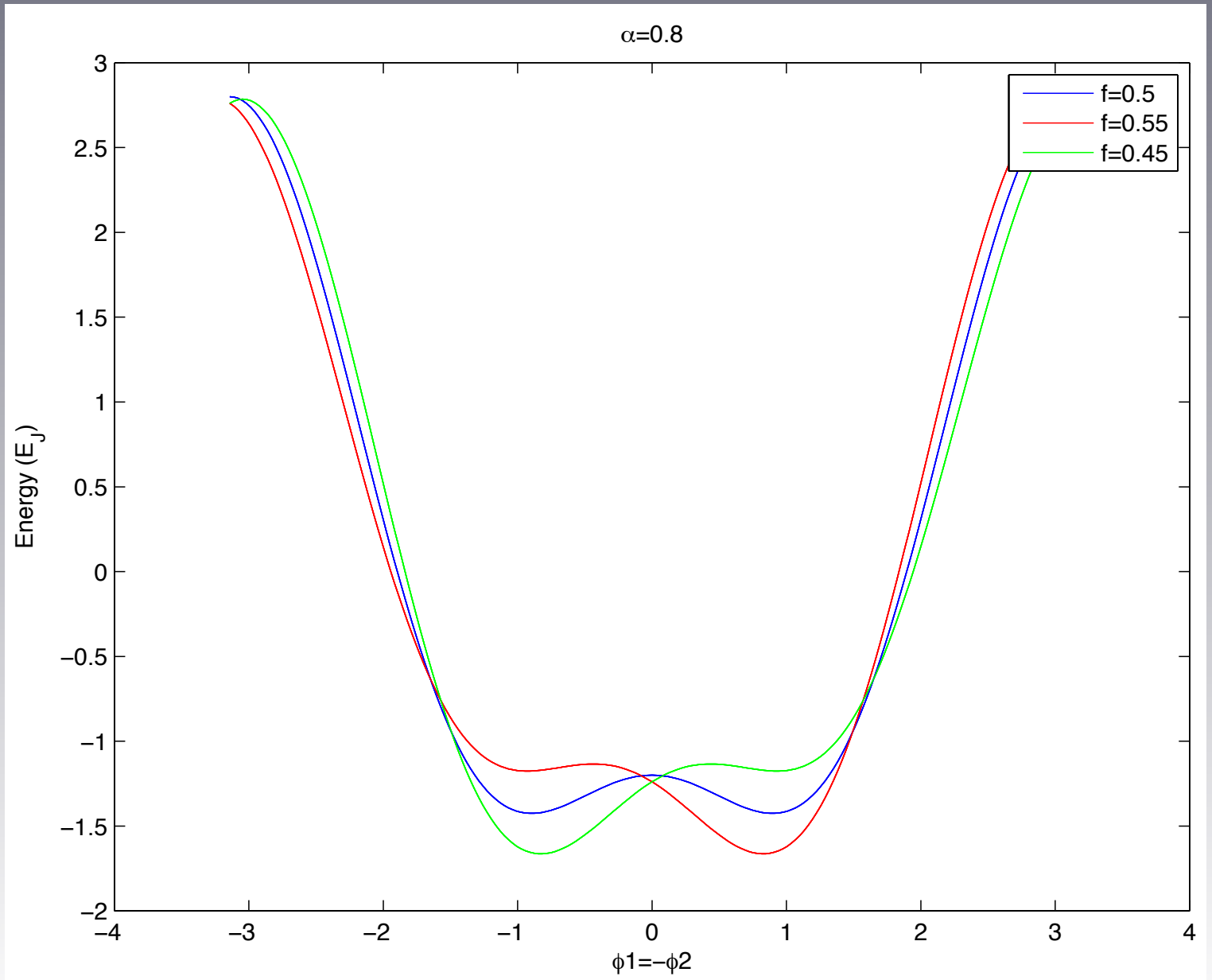
f=0

persistent-current qubit



f=0.5

persistent-current qubit



persistent-current qubit

Classical equations of motion

$$
\begin{aligned}
I_{\text{left}} &= I_0 \sin(\varphi_1) + C\dot{V} = I_0 \sin(\varphi_1) + C\frac{\phi_0}{2\pi}\ddot{\varphi} \\
I_{\text{right}} &= I_0 \sin(\varphi_2) + C\frac{\phi_0}{2\pi}\ddot{\varphi}_2 \\
I_{\text{top}} &= \alpha I_0 \sin(2\pi f - \varphi_1 + \varphi_2) + \alpha C\frac{\phi_0}{2\pi}(\ddot{\varphi}_2 - \ddot{\varphi}_1)
\end{aligned}
$$

$$
I_{\text{left}} = I_{\text{top}} = -I_{\text{right}}
$$

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Lagrangian

$$\begin{aligned}\mathcal{L} &= \mathcal{L}(\varphi_1, \varphi_2, \dot{\varphi}_1, \dot{\varphi}_2) \\ &= \frac{C}{2}\left(\frac{\phi_0}{2\pi}\right)^2 \dot{\varphi}_1^2 + \frac{C}{2}\left(\frac{\phi_0}{2\pi}\right)^2 \dot{\varphi}_2^2 + \alpha\frac{C}{2}\left(\frac{\phi_0}{2\pi}\right)^2 (\dot{\varphi}_1 - \dot{\varphi}_2)^2 \\ &\quad + E_J\cos(\varphi_1) + E_J\cos(\varphi_2) + \alpha E_J\cos(\varphi_1 - \varphi_2 - 2\pi f)\end{aligned}$$

The Lagrange equations reproduce the classical eqations.

$$\frac{d}{dt}\left\{\frac{\partial \mathcal{L}}{\partial \dot{\varphi}_i}\right\} - \frac{\partial \mathcal{L}}{\partial \varphi_i} = 0$$

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From the Lagrangian one can derive the canonical variables

$$
\begin{aligned}
q_1 &= \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_1} = C\left(\frac{\phi_0}{2\pi}\right)^2 \dot{\varphi}_1 + \alpha C\left(\frac{\phi_0}{2\pi}\right)^2 (\dot{\varphi}_1 - \dot{\varphi}_2) \\
q_2 &= \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_2} = C\left(\frac{\phi_0}{2\pi}\right)^2 \dot{\varphi}_2 - \alpha C\left(\frac{\phi_0}{2\pi}\right)^2 (\dot{\varphi}_1 - \dot{\varphi}_2)
\end{aligned}
$$

$$
\begin{aligned}
\dot{\varphi}_1 &= \frac{1}{C\left(\frac{\phi_0}{2\pi}\right)^2} \frac{1+\alpha}{1+2\alpha} q_1 + \frac{1}{C\left(\frac{\phi_0}{2\pi}\right)^2} \frac{\alpha}{1+2\alpha} q_2 \\
\dot{\varphi}_2 &= \frac{1}{C\left(\frac{\phi_0}{2\pi}\right)^2} \frac{\alpha}{1+2\alpha} q_1 + \frac{1}{C\left(\frac{\phi_0}{2\pi}\right)^2} \frac{1+\alpha}{1+2\alpha} q_2
\end{aligned}
$$

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With a Legendre transformation we get the Hamiltonian

$$\mathcal{H}(q_1, q_2, \varphi_1, \varphi_2) = \dot{\varphi}_1 q_1 + \dot{\varphi}_2 q_2 - \mathcal{L}$$

$$\begin{aligned}\mathcal{H} &= 4\frac{E_J}{\left(2e\frac{\phi_0}{2\pi}\right)^2}\left(\frac{1+\alpha}{1+2\alpha}q_1^2 + \frac{2\alpha}{1+2\alpha}q_1 q_2 + \frac{1+\alpha}{1+2\alpha}q_2^2\right) \\ &\quad -E_J\cos(\varphi_1) - E_J\cos(\varphi_2) - \alpha E_J\cos(\varphi_1 - \varphi_2 - 2\pi f)\end{aligned}$$

From the canonical equations

$$\dot{\varphi}_i = \frac{\partial \mathcal{H}}{\partial q_i}$$

$$\dot{q}_i = -\frac{\partial \mathcal{H}}{\partial \varphi_i}$$

one gets back the classical equations of motion (4).

persistent-current qubit

Qubit Hamiltonian in the Charge Base

using

$$
\begin{aligned}
q_1 &= 2e\frac{\phi_0}{2\pi}n_1 \\
q_2 &= 2e\frac{\phi_0}{2\pi}n_2
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{H} &= 4E_c\frac{1+\alpha}{1+2\alpha}n_1^2 + 4E_c\frac{2\alpha}{1+2\alpha}n_1n_2 + 4E_c\frac{1+\alpha}{1+2\alpha}n_2^2 \\
&\quad -E_J\cos(\varphi_1) - E_J\cos(\varphi_2) - \alpha E_J\cos(\varphi_1-\varphi_2-2\pi f)
\end{aligned}
$$

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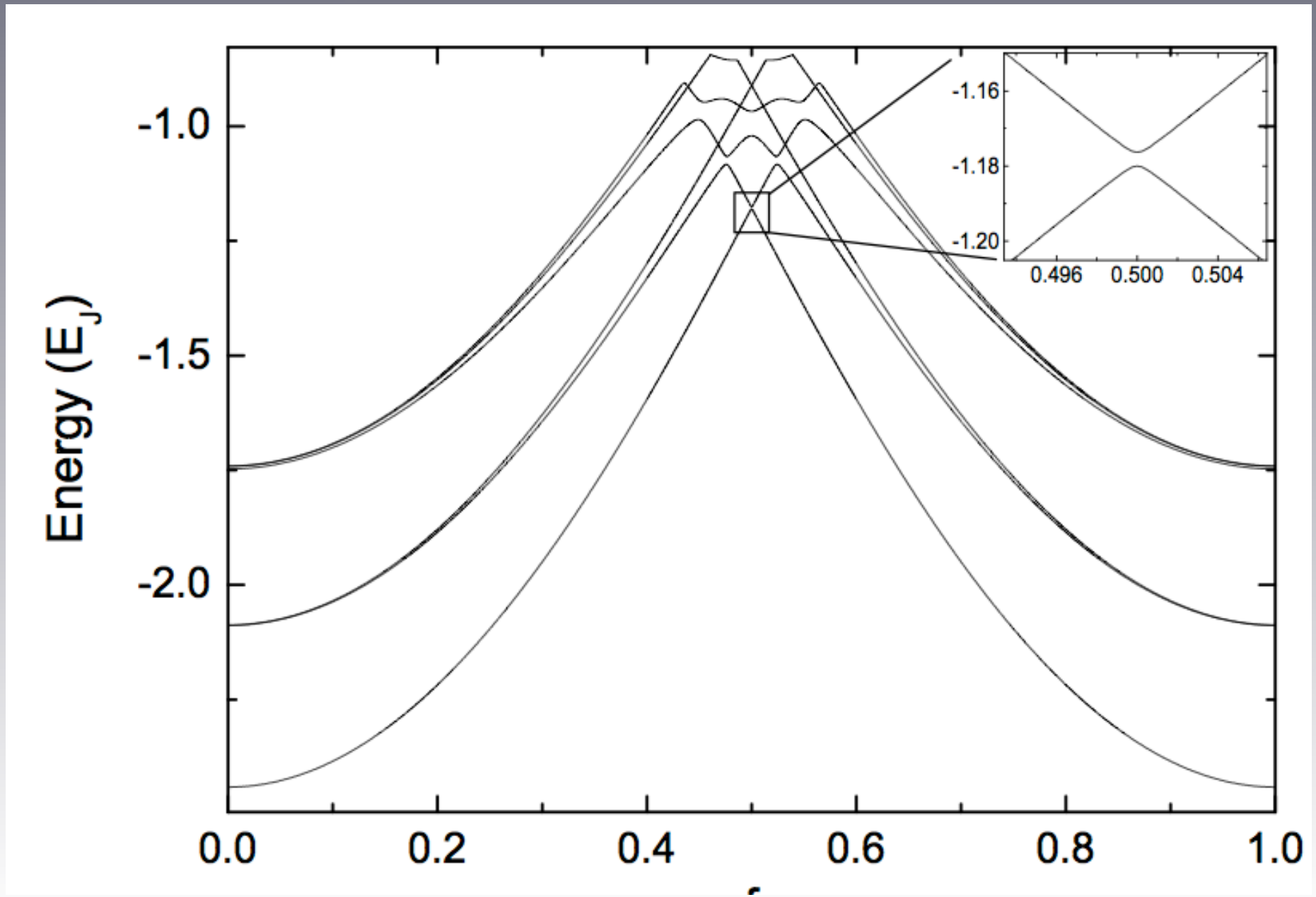
$$
\begin{aligned}
e^{i\varphi_j}\,|n_j\rangle &= |n_j+1\rangle \\
e^{-i\varphi_j}\,|n_j\rangle &= |n_j-1\rangle \\
e^{i(\varphi_j+k)}\,|n_j\rangle &= e^{ik}\,|n_j+1\rangle \\
e^{i(\varphi_j-k)}\,|n_j\rangle &= e^{-ik}\,|n_j-1\rangle
\end{aligned}
$$

$$
\langle m_1,m_2|-E_J\cos(\varphi_1)\,|n_1,n_2\rangle = \delta_{m_2,n_2}\frac{-E_J}{2}(\delta_{m_1,n_1-1}+\delta_{m_1,n_1+1})
$$

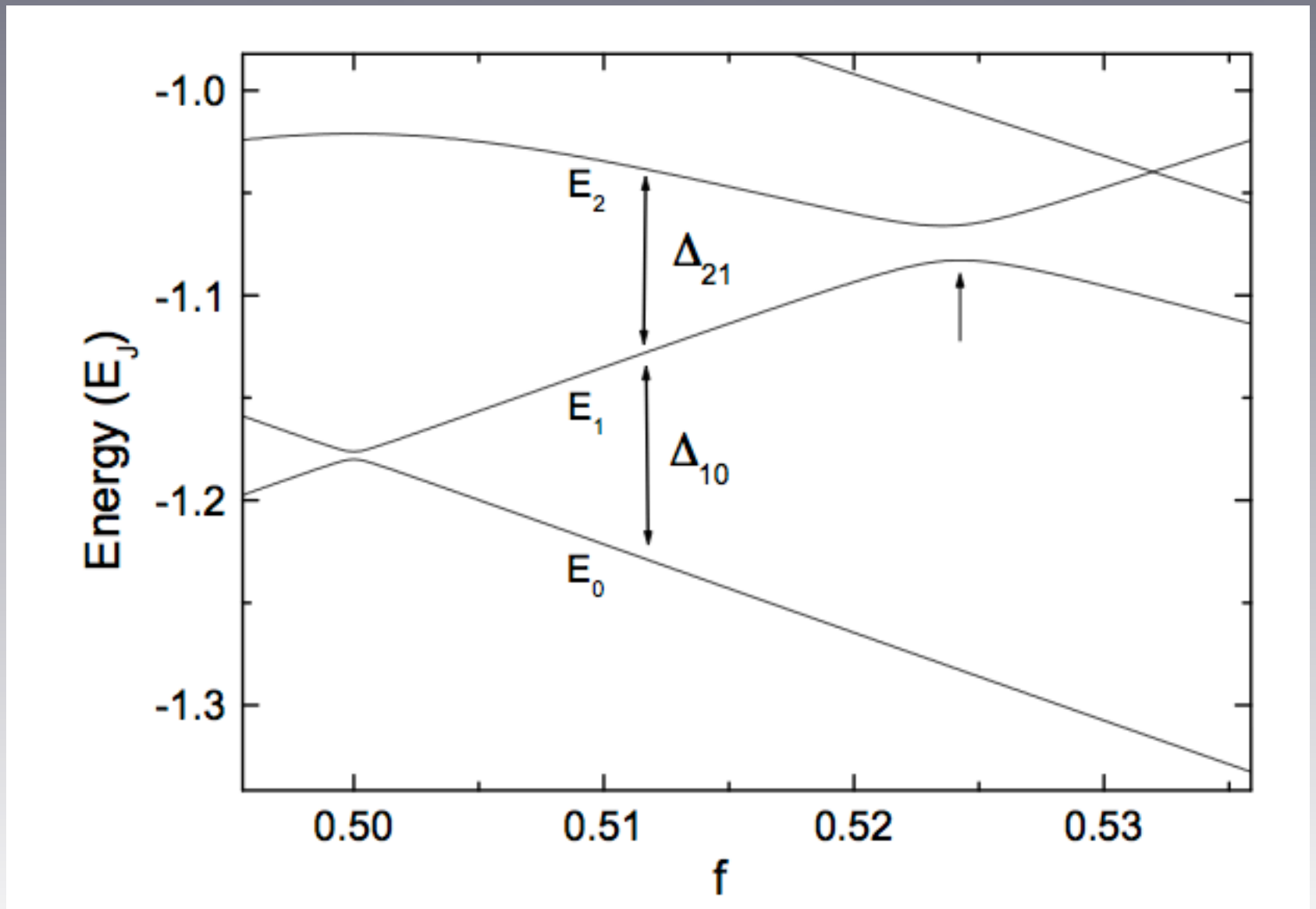
$$
\langle m_1,m_2|-E_J\cos(\varphi_2)\,|n_1,n_2\rangle = \delta_{m_1,n_1}\frac{-E_J}{2}(\delta_{m_2,n_2-1}+\delta_{m_2,n_2+1})
$$

$$
\begin{aligned}
&\langle m_1,m_2|-\alpha E_J\cos(\varphi_1-\varphi_2-2\pi f)\,|n_1,n_2\rangle = \\
&\left(\delta_{m_1,n_1+1}\delta_{m_2,n_2-1}e^{-i2\pi f}+\delta_{m_1,n_1-1}\delta_{m_2,n_2+1}e^{i2\pi f}\right)\frac{-\alpha E_J}{2}
\end{aligned}
$$

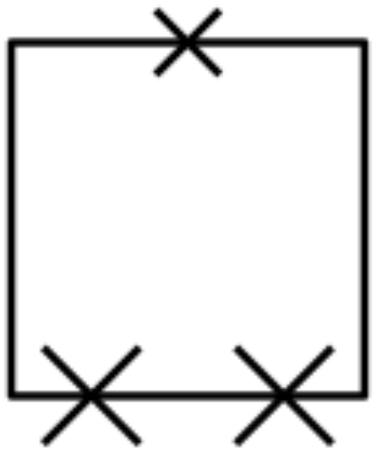
persistent-current qubit

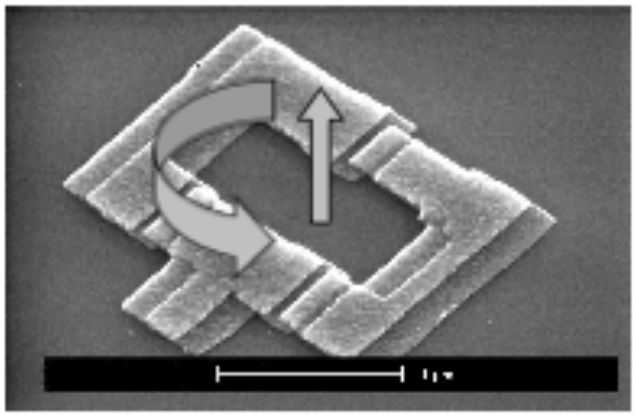


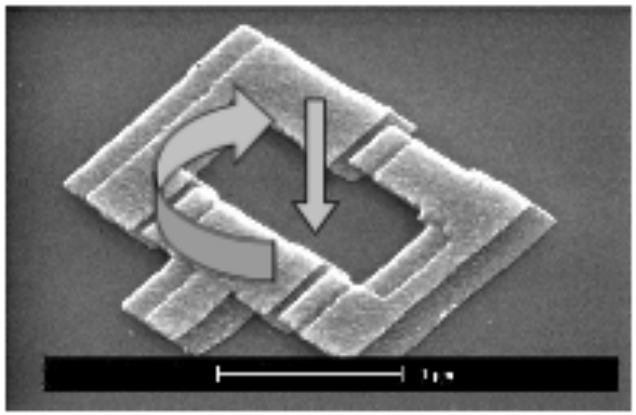
persistent-current qubit



persistent-current qubit







spin up

spin down

persistent-current qubit

Quantum Superposition of Macroscopic Persistent-Current States

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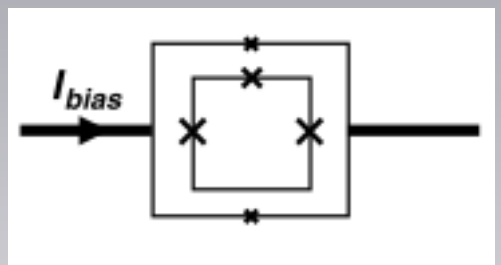
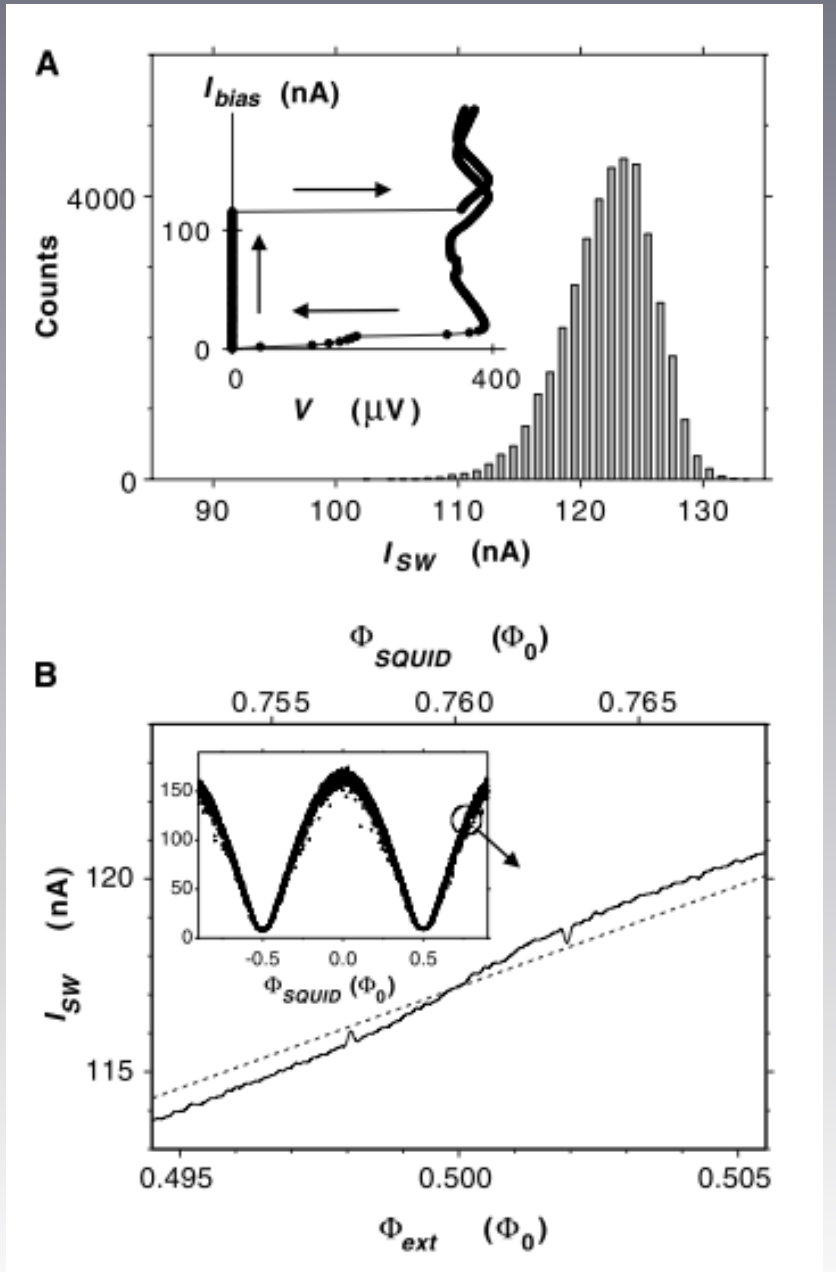
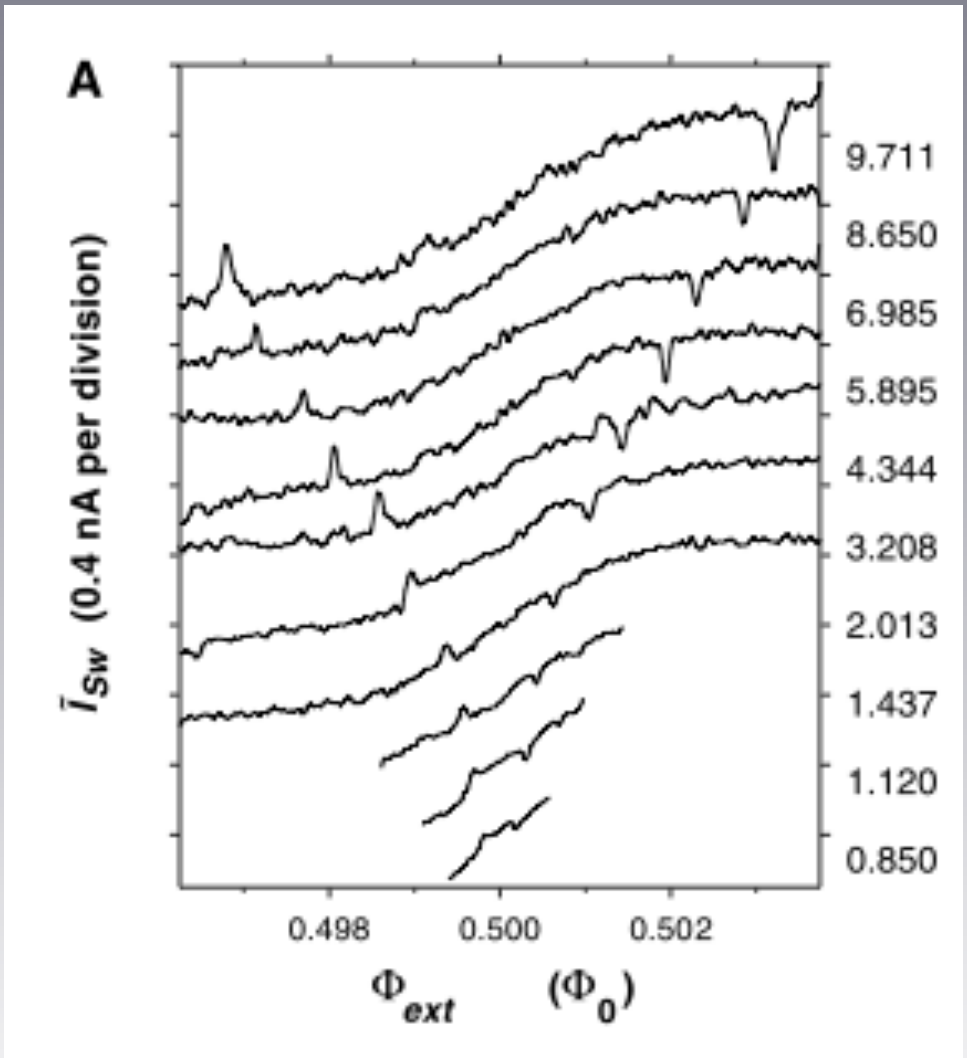
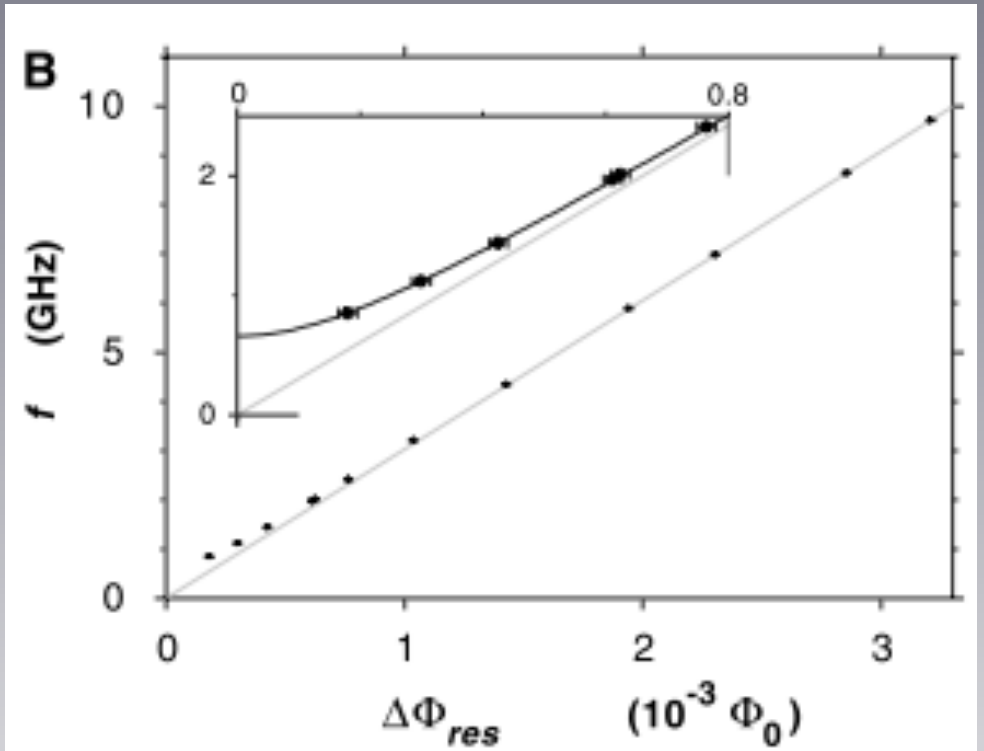
R. N. Schouten,[1] C. J. P. M. Harmans,[1] T. P. Orlando,[2]
Seth Lloyd,[3] J. E. Mooij[1,2]

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